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Memory Function for Dressed Particles

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This paper is concerned with the calculation of the memory function and derivation of a kinetic equation for one-body phase space correlation functions. The theory uses a one-body additive projection operator and a division of the Liouville operator with an unperturbed part that describes dressed particles. Binary collisions are neglected, for the theory aims at describing the screening and backflow effects of a type contained in the plasma kinetic theory of Balescu and Lenard. We obtain an explicit kinetic equation which is an improvement of these theories for the plasma case, and involves the exact equilibrium pair and triplet distributions. The equation also describes systems with strong short-range forces and shows how the screening effects occur in this case as well. The unifying function is the direct correlation function. The theory is meant to provide understanding for a more complete theory of fluids where a proper account is given of close collisions.

KEY WORDS: Memory function; kinetic equation; dressed particles.

1. INTRODUCTION

The present work is concerned with the calculation of time-dependent correlation functions in the linear response domain. The time correlations are of the form

$$C(\mathbf{pk}|\mathbf{p'k}) \equiv \int d\Gamma\ \Phi\hat{N}^*(\mathbf{p}|\mathbf{k}|0)\hat{N}(\mathbf{p'}|\mathbf{k}|t) \equiv \langle\hat{N}(\mathbf{p}|\mathbf{k}|0)|\hat{N}(\mathbf{p'}|\mathbf{k}|t)\rangle \qquad (1)$$

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Here

$$\hat{N}(\mathbf{p}|\mathbf{k}|t) = \sum_{\alpha} \delta(\mathbf{p}_{\alpha}(t) - \mathbf{p}) \exp[i\mathbf{k}\mathbf{q}_{\alpha}(t)] \tag{2}$$

is the one-body additive phase space distribution. Φ is the Gibbs equilibrium distribution and an inner product in phase space is defined as indicated.

There has been a vast amount of study of this problem in recent years.[2] In addition to the new results presented here, we point out some not so obvious connnections between some of the theories. We start with an abstract operator formulation of the standard Mori memory function formalism.[3] This formalism involves the choice of a projection operator P. It may be chosen so as to project out a limited number of phase space functions of a given wave vector $\mathbf{k}$, as, for example, the hydrodynamic states used in the theories of Kadanoff and Swift[4] and Kawasaki.[5] Alternatively, as was shown by Akcasu and Duderstadt,[6] P may be taken to be the entire space of one-body additive functions. The projection operator can still be exhibited in closed form. In any case, the formalism for calculating correlation functions involves a "static" part PLP, where L is the Liouville operator. This is Mori's frequency matrix. This term involves the total Liouville operator and is independent of any breakup of L into parts. If P is chosen to project onto the space spanned by the five hydrodynamic states for given wave vector $\mathbf{k}$, one is concerned with diagonalizing the Liouville operator in this finite basis (cf. Zwanzig and Nossal[7]. This gives a microscopic formulation of reversible, perfect fluid theory. If, on the other hand, one takes P to be constructed from the infinite set of one-body additive functions, one finds the Zwanzig variant of the linearized Vlasov equation.

In both cases the other part of the formalism, involving the memory function, has been treated by simply setting the memory function equal to zero.

At this point we briefly set down the equations of the memory operator formalism. We work with the Laplace transform of the correlation function

$$\bar{C}(\mathbf{p}\mathbf{k}|\mathbf{p}'\mathbf{k}) = \langle N(\mathbf{p}|\mathbf{k})|G|N(\mathbf{p}'|\mathbf{k})\rangle \tag{3}$$

where the resolvent operator G is

$$G \equiv (s + L)^{-1} \tag{4}$$

The resolvent operator applied to the initial value of $\hat{N}(\mathbf{p}'|\mathbf{k}|t)$, i.e., $N(\mathbf{p}'|\mathbf{k}|t = 0) \equiv N(\mathbf{p}'|\mathbf{k})$, yields the Laplace transform of $\hat{N}(\mathbf{p}'|\mathbf{k}|t)$.

We are therefore concerned with the calculation of PGP, where P is some projection operator. From

$$(s + L)G = 1 \tag{5}$$

[2] See, e.g., Berne.[1] For an extensive set of references see Mazenko.[15] Also see Lebowitz *et al.*[2]

we obtain by applying first P, then Q,

$$(s + PLP)PG + PLQG = P, \qquad (s + QLQ)QG + QLPG = Q \tag{6}$$

where

$$Q = 1 - P, \qquad P^2 = P \tag{7}$$

Each equation breaks into two parts when P and Q are applied on the right-hand side. From the second equation, the P operator yields

$$(s + QLQ)QGP = -(QLP)PGP \tag{8}$$

Inserting this into the equation

$$(s + PLP)PGP + PLQ \cdot QGP = P^2 = P \tag{9}$$

we find

$$(s + PLP + M)(PGP) = P \tag{10}$$

where $\overline{M}$ is the memory operator

$$\overline{M} = -PLQ(s + QLQ)^{-1}QLP \tag{11}$$

In this formalism the theories already mentioned (perfect fluid, modified Vlasov) correspond to different choices of P and total neglect of the memory function. The two-body additive approximation of Gross[8] can be characterized as the choice of P as a projection onto the class of two-body additive functions in phase space. The memory function is entirely neglected. A detailed analysis of this operator is found in the paper by Bergeron *et al.*[9] under the title, "Inversion Problem." Of course, when P consists of a finite number of functions (for each wave vector), one obtains reversible equations, i.e., a finite number of oscillating terms for the correlation function. With the one-body additive choice, one obtains the analog of Landau damping, while with the two-body additive projection operator one has Boltzmann–Enskog and Balescu–Lenard[10] type kinetic equations for the correlation functions. Thus, one approach to a systematic theory can be characterized by the injunction, Enlarge the projection operator space and neglect the residual memory operator. Another approach is to try to estimate the memory function intuitively (e.g., Kadanoff and Swift) or to try to calculate by some systematic scheme such as perturbation theory. The approach is similar to the mass operator approach of quantum field theory. Thus Akcasu and Duderstadt consider the case that P is the one-body additive operator and the memory function is approximated as

$$M \approx -PLQ(s + QL_0{}^0Q)^{-1}QLP$$

Here $L_0{}^0$ is the free streaming part of the Liouville operator and $L_1{}^0$ is the potential energy part. Since $L_0{}^0$ is a one-body additive operator, $L_0{}^0P$ is a

one-body additive function and $QL_0{}^0P = 0$ ($PL_0{}^0Q$ is not zero). This is an attractive feature of P. The resulting theory is then that of Forster and Martin,[11] which is a non-Markovian extension of the weak coupling Fokker–Planck kinetic equation for the correlation function.

In contrast to the "static" term PLP, which depends only on P, the approximation to the memory function is sensitive to the division of L into an L_0 and an L_1. The main calculation of this paper involves the computation of the memory function in the manner just outlined but with L_0 chosen to be a different one-body additive operator in which a self-consistent field term, depending on the equilibrium direct correlation function, is added to the free streaming operator. The residual operator L_1 has the property that $PL_1P = 0$, i.e.,

$$\langle N(\mathbf{p}|\mathbf{k})|L_1|N(\mathbf{p}'|\mathbf{k})\rangle = 0$$

for all values of the arguments.

The theory that results when one performs this simple step of working with "dressed" propagators has some attractive features. When applied to the Coulomb case, with equilibrium correlation functions estimated in the Debye–Hückel approximation, one obtains the standard linearized plasma theory of Balescu, Lenard, and Guernsey (see Ref. 12) together with higher order corrections. The Fokker–Planck-like equations have screening functions that avoid long-wavelength divergences. Of course, even if one is in a physical domain where the plasma parameter, i.e., the reciprocal of number of electrons in a Debye sphere, is small, the Debye–Hückel approximation fails at short distance. The present theory is free of divergences arising from this fact since we work with the exact equilibrium correlation functions. The physical analysis of these screening-type terms in terms of "dressed particles," "deformed polarization clouds," "hydrodynamic backflow" is well known in plasma physics.[10] Since our theory also applies to neutral systems with short-range forces, we have the analogous microscopic description of effects that enter in the classical hydrodynamics of the interaction of macroscopic bodies in a fluid.

Clearly, the fact that the theory does not lead to a nonsensical description of short-wave interactions does not mean that it leads to a correct description. Indeed, we know that the description is not correct, since we have neglected the direct short-range binary interactions that lead to the Boltzmann–Enskog equation. For the charged particle case, the direct Coulomb interaction is usually dropped on the grounds that it is of higher order in the plasma parameter. It is this neglect that makes possible the construction of an explicit equation obeyed by the singlet distribution function, e.g., Guernsey's[12] application of singular integral equation theory. For neutral systems with short-range forces one can say that the Forster–Martin

theory is superior to the earlier Brout–Prigogine (see, e.g., Ref. 13) weak coupling theory in the same manner. The Forster–Martin theory involves equilibrium correlation functions in a way that the theory is meaningful even for very strong short-range forces. Yet it is not correct since no attempt was made to treat binary collisions correctly.

One knows that theories based on a two-body additive projection operator do give the Boltzmann–Enskog limit. But of course the theories are much more complicated. The point of the present paper is to show that for a given projection operator (in this case the one-body additive operator) significant progress can be made in estimating the memory operator by an appropriate division of the Liouville operator. The calculations are presented in Sections 3–5.

Section 2 is devoted to the analysis of an interesting trap that one can fall into when working with memory operators. The author fell into the trap while analyzing a very interesting recent paper by Akcasu.[14] We hope the analysis will prevent some others from having the same experience.

2. INTEGRAL EQUATIONS FOR THE MEMORY FUNCTION

The exact expression for the memory operator involves the resolvent $(s + QLQ)^{-1}$, i.e., the projection of the Liouville operator in the Q space. It is inconvenient to calculate with such propagators. Following Mori and Akcasu,[14] one can use the algebraic identity

$$(s + QL)^{-1} = G + GPL(s + QL)^{-1}, \qquad G = (s + L)^{-1} \tag{12}$$

to form an equation for the memory operator. To simplify the argument, let us take the division $L = L_0 + L_1$, such that L_0 is one-body additive. We take P to be the one-body projection operator. Then $QL_0P = 0$ and

$$M = q + \Lambda \cdot M \tag{13}$$

Here

$$q = -PLGQL_1P \tag{14}$$

$$\Lambda = PLGP \tag{15}$$

The important point is that q and Λ contain the normal resolvent operator G. If one imagines that suitable approximations are made for q and Λ, we have an integral equation for the memory function. The formal solution is

$$M = (1 - \Lambda)^{-1}q \tag{16}$$

However, there is an algebraic relation connecting q and Λ. Using

$$G = G_0 - GL_1G_0, \qquad G_0 = (s + L_0)^{-1} \tag{17}$$

we find

$$\Lambda = \Lambda^0 - \Lambda \cdot \Lambda^0 + qPG_0P + PL_0G_0P, \qquad \Lambda^0 \equiv PL_1G_0P \tag{18}$$

G_0 and Λ^0 can be treated as known quantities.

It is now tempting to try to generate the entire theory from q, the correlation function of "random force." We can compute Λ by solving the integral equation formally

$$\Lambda = \{\Lambda^0 + PL_0G_0P + qPG_0P\}(1 + \Lambda^0)^{-1} \tag{19}$$

Inserting this into the integral equation for the memory function, we find the exact algebraic result

$$M = (1 + \Lambda^0)[1 - PL_0G_0P - qPG_0P]^{-1}q \tag{20}$$

With an approximation to q we find corrections to the memory function that depend on higher powers of q. However, the point made here is that this is devoid of content. The memory function integral equation just "undoes" the whole memory function approach. It is easiest to see this in the spatially homogeneous case. Then P only contains the zero-wavevector, one-body additive functions of velocity. We have

$$PG_0P = 1/s, \qquad PLP = 0 \qquad \Lambda^0 = 0 \tag{21}$$

Thus, Eq. (20) gives for the memory function

$$M = [1 - q/s]^{-1}q \tag{22}$$

The equation for the correlation function is

$$[s + (1 - q/s)^{-1}q]PGP = P \tag{23}$$

These results depend on exact algebraic manipulations and do not depend on using the exact q or some approximate estimate for q. We see that for the spatially homogeneous problem

$$PGP = (1/s) - (1/s^2)q \tag{24}$$

This is of course an exact identity that can be established directly from the definition of q. The significant point is that even if we use an approximation for q but solve the integral equation for the memory function exactly, we arrive at an approximation to PGP of the above form. The structure of PGP in the complex s plane thus mirrors precisely that assumed in the (usually crude) estimate of q. This is the sense in which one has "undone" the memory function.

It is, however, perfectly valid to use the integral equation for the memory function as a calculational aid, as is done in Akcasu's paper. If one believes that it is adequate to have the memory function to some order in the coupling constant or density (for all values of s and k), it may be easier to evaluate q,

since it contains the ordinary resolvent G. One can then calculate the memory operator to the requisite accuracy.

We now complete the above argument for the general, spatially inhomogeneous case in a way that is germane to the rest of this paper. P is the one-body additive projection operator and $PL_1P = 0$. Then $\Lambda^0 = 0$,

$$M = \{(s - q)PG_0P\}^{-1}q \tag{25}$$

This leads to

$$\{s + PL_0P + M\}PGP = P \tag{26}$$

and

$$PGP = (1 - q/s)(PG_0P) \tag{27}$$

In the light of the above considerations we will simply approximate M by q, without solving the integral equation, and justify the results as in a coupling constant expansion. To improve the theory, we need to enlarge the space spanned by P, or to find a systematic approach to M.

The most systematic approach to memory functions is the impressive set of papers by Mazenko.[15] The relation between different recent approaches has been clarified by the work of Boley[16] on extending the present author's approach to nth approximation.

We can, however, easily show that the present work is in an approximation that is intermediate between the one-body additive and two-body additive approximations. To translate the two-body additive approximation into the memory function language, we let Q be a projection operator constructed from two-body additive functions but made orthogonal to the one-body projector P. The rest of the function space is neglected. Then QLP consists of at most two-body additive functions. The propagator $[s + QLQ]^{-1}$ involves the matrix elements in the two-body space QLQ. We will replace this by $[s + QL_0Q]^{-1}$ with a suitable L_0. The L_0 describes the propagation of independent "dressed" particles in the complete two-body additive space $P + Q$. Thus $[s + QL_0Q]^{-1}$ represents such independent dressed particles but subject to an orthogonality constraint that arises from the particle nature involved in excitations. What is neglected is the explicit binary collisions that are contained in the full QLQ, i.e., in the two-body additive approximation. This kind of approximation is familiar in plasma physics.

3. DIVISION OF THE LIOUVILLE OPERATOR

We first set down the formalism that will be used in the remainder of this paper. The Liouville operator is

$$L = \sum_{\alpha=1}^{N} \frac{\mathbf{p}_\alpha}{m} \frac{\partial}{\partial \mathbf{q}_\alpha} - \sum_{\alpha} \frac{\partial V}{\partial \mathbf{q}_\alpha} \frac{\partial}{\partial \mathbf{p}_\alpha} = L_0{}^0 + L_1{}^0 \tag{28}$$

The matrix elements of L between one-body additive functions are

$$\langle N(\mathbf{p}'|\mathbf{k})|L|N(\mathbf{p}|\mathbf{k})\rangle = i(\mathbf{k}\cdot\mathbf{p}/m)\phi(p)N\,\delta(\mathbf{p}-\mathbf{p}') \tag{29}$$
$$\phi(p) = (\theta/2\pi m)^{3/2}\exp(-\theta p^2/2m)$$

In addition, the separate parts have matrix elements

$$\langle N(\mathbf{p}'|\mathbf{k})|L_0{}^0|N(\mathbf{p}|\mathbf{k})\rangle = i(\mathbf{k}\cdot\mathbf{p}/m)\phi(p)[N\,\delta(\mathbf{p}-\mathbf{p}') + r(k)\phi(p')] \tag{30}$$
$$\langle N(\mathbf{p}'|\mathbf{k})|L_1{}^0|N(\mathbf{p}|\mathbf{k})\rangle = -i(\mathbf{k}\cdot\mathbf{p}/m)\phi(p)r(k)\phi(p')$$

Here

$$r(k) \equiv \int \phi \sum_{\alpha\neq\beta} e^{ik(q_\alpha - q_\beta)}\,d\Gamma \tag{31}$$

is the Fourier transform of the pair correlation function.

The one-body projection operator can be written as

$$P = \int\int \sum_k |N(\mathbf{p}|\mathbf{k})\rangle\, d^3p\, Z(\mathbf{p},\mathbf{k}|\mathbf{p}',\mathbf{k})\, d^3p'\, \langle N(\mathbf{p}'|\mathbf{k})| \tag{32}$$

with

$$NZ(\mathbf{p},\mathbf{k}|\mathbf{p}',\mathbf{k}) = \phi^{-1}(p)\,\delta(\mathbf{p}-\mathbf{p}') - \{r(k)/[N + r(k)]\} \tag{33}$$

Since

$$\langle N(\mathbf{p}|\mathbf{k})|N(\mathbf{p}'|\mathbf{k})\rangle = N\phi(p)\{\delta(\mathbf{p}-\mathbf{p}') + [r(k)/N]\phi(p')\} \tag{34}$$

one checks that

$$P|N(\mathbf{p}|\mathbf{k})\rangle = |N(\mathbf{p}|\mathbf{k})\rangle$$

The desired correlation function is

$$\bar{C}(\mathbf{p},\mathbf{k}|\mathbf{p}',\mathbf{k}) = \langle N(\mathbf{p}|\mathbf{k})|PGP|N(\mathbf{p}'|\mathbf{k})\rangle \equiv \langle\mathbf{p}|PGP|\mathbf{p}'\rangle \equiv \bar{C}(\mathbf{p}|\mathbf{p}') \tag{35}$$

Since the wave vector $\mathbf{k}$ is a parameter, we use the simpler notation where only the momentum part is indicated. We find

$$s\bar{C}(\mathbf{p}|\mathbf{p}') + \int\int \langle\mathbf{p}|L + \bar{M}(s)|\mathbf{p}''\rangle\, d^3p''\, Z(\mathbf{p}''|\mathbf{p}_1)\bar{C}(\mathbf{p}_1|\mathbf{p}')\, d^3p_1$$
$$= \langle N(\mathbf{p}|\mathbf{k})|N(\mathbf{p}'|\mathbf{k})\rangle \tag{36}$$

This is the matrix realization of the basic equation (10). We note that the second part of the projection operator is a constant and involves a contribution of $\langle N(\mathbf{p}|\mathbf{k})|\bar{M}|\rho(\mathbf{k})\rangle$ in the memory part of the equation. However, $L_1{}^0\rho(\mathbf{k}) = 0$. The memory operator contains the factor $QL_1{}^0P$ on the right-

hand side. This remains true in all of our approximations to the memory operator. We can therefore rewrite the equation in the form

$$\left(s + i\frac{\mathbf{k}\cdot\mathbf{p}}{m}\right)\bar{C}(\mathbf{p}|\mathbf{p}') - i\frac{\mathbf{k}\cdot\mathbf{p}}{m}\phi(p)\frac{r}{N+r}\int \bar{C}(\mathbf{p}_1|\mathbf{p}')\,d^3p_1 + \int \langle\mathbf{p}|\bar{M}|\mathbf{p}_1\rangle \frac{d^3p_1}{N\phi_1}\bar{C}(\mathbf{p}_1|\mathbf{p}') = \langle N(\mathbf{p}|\mathbf{k})|N(\mathbf{p}'|\mathbf{k})\rangle \tag{37}$$

The "static" part of the equation corresponds to the Zwanzig modification of the linearized Vlasov equation. The Forster–Martin weak coupling theory consists in taking

$$\bar{M} \approx \bar{M}^F = -PL_1{}^0Q[s + QL_0{}^0Q]^{-1}QL_1{}^0P \tag{38}$$

$\bar{M}^F$ stands for the free streaming memory function. We find for spatially homogeneous processes

$$\langle N(\mathbf{p}'|0)|\bar{M}^F|N(\mathbf{p}'|0)\rangle = -\frac{1}{\theta}\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial p_{\mu'}}\sum k_\nu k_\mu \frac{\tilde{V}(k)}{\Omega} r(k) \times \left[\delta(\mathbf{p}-\mathbf{p}')B_{-k}(\mathbf{p}) - \frac{\phi(p')}{s + (i\mathbf{k}/m)(\mathbf{p}-\mathbf{p}')}\right]\phi(p) \tag{39}$$

where

$$B_k(\mathbf{p}) = \int d^3p'\,\phi(p')[s + (i\mathbf{k}/m)(\mathbf{p}-\mathbf{p}')]^{-1}$$

As noted earlier, it differs from the Brout–Prigogine weak coupling memory function in that $r(k)$ is not replaced by its weak coupling equilibrium value $-\rho_0\theta\tilde{V}(k)(N-1)$.

We now discuss the different splitting of the Liouville operator that is used in this paper. It is convenient to use a functional notation. The symmetric N-particle distribution function $\bar{F}(\mathbf{q}_1, \ldots, \mathbf{p}_N|s)$ is taken to depend on the particle variables through their appearance in the functions $N(\mathbf{p}|\mathbf{k})$. $\bar{F}$ is a superposition of products of the $N(\mathbf{p}_1|\mathbf{k}_1)\cdots N(\mathbf{p}_N|\mathbf{k}_N)$ with different values of $\mathbf{p}_1, \ldots, \mathbf{p}_N, \mathbf{k}_1, \ldots, \mathbf{k}_N$.

Then

$$L_0{}^0\bar{F} = \int \sum_{\alpha=1}^{N}\frac{\mathbf{p}_\alpha}{m}\sum_k \frac{\partial N(\mathbf{p}|\mathbf{k})}{\partial \mathbf{q}_\alpha}\frac{\delta}{\partial N(\mathbf{p}|\mathbf{k})}\,d^3p\bar{F} = \int d^3p \sum_k \frac{i\mathbf{k}\cdot\mathbf{p}}{m}N(\mathbf{p}|\mathbf{k})\frac{\delta\bar{F}}{\delta N(\mathbf{p}|\mathbf{k})} \tag{40}$$

We also have

$$L_1{}^0\bar{F} = \frac{-i}{\Omega}\int d^3p \sum_{k',k} \mathbf{k}'\tilde{V}(k')\rho(\mathbf{k}')\frac{\partial N(\mathbf{p}|\mathbf{k}-\mathbf{k}')}{\partial \mathbf{p}}\frac{\delta \bar{F}}{\delta N(\mathbf{p}|\mathbf{k})} \tag{41}$$

$$\rho(\mathbf{k}) = \int N(p_1|\mathbf{k})\, d^3p_1 \tag{42}$$

The standard Vlasov random phase term is obtained when one selects the $\mathbf{k}' = \mathbf{k}$ term from $L_1{}^0$ and replaces $\partial N(p|0)/\partial p$ by $N\,\partial\phi(p)/\partial \mathbf{p}$. Our procedure is to isolate a term of the same form. Thus we write

$$L_0 = \int d^3p \sum_k \left\{\frac{i\mathbf{k}\cdot\mathbf{p}}{m} N(\mathbf{p}|\mathbf{k}) - i\frac{\mathbf{k}\tilde{\Psi}(k)}{\Omega}\frac{N\,\partial\phi(p)}{\partial \mathbf{p}}\rho(\mathbf{k})\right\}\frac{\delta}{\delta N(\mathbf{p}|\mathbf{k})} \tag{43}$$

$$L_1 = L - L_0$$

The additional term in L_0 is one-body additive, i.e., linear in the $N(\mathbf{p}_1|\mathbf{k})$ through $\rho(k)$. It is in fact an integral operator in phase space. We will choose

$$\tilde{\Psi}(k) = \frac{-r(k)}{N + r(k)}\frac{1}{\theta} \tag{44}$$

i.e., the Fourier transform of the direct correlation function. The matrix element $\langle N(\mathbf{p}|\mathbf{k})|L_0|N(\mathbf{p}'|\mathbf{k})\rangle$ gives the Zwanzig term in the integral equation for the correlation function. We will refer to the propagator $(s + L_0)^{-1}$ as a "dressed propagator." The remaining part of the Liouville operator is

$$L_1 = L_1{}^0 + i\sum_k \int d^3p \frac{\mathbf{k}\tilde{\Psi}(k)}{\Omega}\frac{N\,\partial\phi(p)}{\partial \mathbf{p}}\rho(\mathbf{k})\frac{\delta}{\delta N(\mathbf{p}|\mathbf{k})} \tag{45}$$

It has the property that $\langle N(\mathbf{p}|\mathbf{k})|L_1|N(\mathbf{p}'|\mathbf{k})\rangle = 0$. This follows from the equilibrium chain for the distribution functions. Indeed, in the functional language one fixes $\tilde{\Psi}(k)$ by adding to $L_1{}^0$ a term linear in the $N(\mathbf{p}|\mathbf{k})$ and in the functional derivatives. One requires the vanishing of the matrix elements of L_1 in the one-body space.

The operator L_0 has a very important property which can be termed "additivity of excitations." If $\psi_1(\mathbf{k}_1)$ and $\psi_2(\mathbf{k}_2)$ are eigenfunctions of L_0 with eigenvalues $\epsilon(\mathbf{k}_1)$ and $\epsilon(\mathbf{k}_2)$, the product $\psi_1\psi_2$ is an eigenfunction with eigenvalue $\epsilon(\mathbf{k}_1) + \epsilon(\mathbf{k}_2)$. This means that $(s + L_0)^{-1}$ creates dressed excitations in the multiparticle Q space. This is a crucial property that depends on having a differential operator in L. One can easily construct an L_1 by writing $L_1 = L_1{}^0 - PL_1{}^0P$ and $L_0 = L_0{}^0 + PL_1{}^0P$ with the property $PL_1P = 0$. However, the additional (integral) operator acts only in the P space and $(s + L_0)^{-1}$ is only a free streaming propagator in Q space.

Our problem is now well formulated. We will calculate the memory function in the approximation

$$\langle \mathbf{p}|\bar{M}(s)|\mathbf{p}'\rangle = -\langle N(\mathbf{p}|\mathbf{k})|L_1{}^0Q(s + QL_0Q)^{-1}QL_1{}^0|N(\mathbf{p}'|\mathbf{k})\rangle \tag{46}$$

4. EVALUATION OF MEMORY FUNCTION FOR DRESSED PARTICLES

To evaluate $\overline{M}(s)$, note that

$$Q(s + QL_0Q)^{-1}Q = Q\{(s + L_0)^{-1} + (s + L_0)^{-1}PL_0(s + QL_0)^{-1}\}Q \quad (47)$$

Since $Q(s + L_0)^{-1}P = 0$, $QLP = QL_1{}^0P$, we have

$$\overline{M}(s) \approx -PLQ(s + L_0)^{-1}QLP \quad (48)$$

It is easier to calculate directly in the time domain,

$$\hat{M}(t) = -PLQe^{-L_0t}L_1{}^0P \quad (49)$$

We are concerned with the matrix elements

$$\begin{aligned}\langle \mathbf{p}'|M(t)|\mathbf{p}\rangle &= -\langle N(\mathbf{p}'|\mathbf{k})|LQe^{-L_0t}L_1{}^0|N(\mathbf{p}|\mathbf{k})\rangle \\ &= \langle L_1{}^0N(\mathbf{p}'|\mathbf{k})|Qe^{-L_0t}L_1{}^0|N(\mathbf{p}|\mathbf{k})\rangle \end{aligned} \quad (50)$$

where the last equation is a consequence of $L\Phi = 0$. Thus the memory function starts as second order in the coupling constant when $t = 0$.

We now have

$$L_1{}^0|N(\mathbf{p}|\mathbf{k})\rangle = \frac{-i}{\Omega}\frac{\partial}{\partial \mathbf{p}}\sum \mathbf{k}_1\tilde{V}(k_1)|\rho(\mathbf{k}_1)N(\mathbf{p}|\mathbf{k}_2)\rangle, \qquad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1 \quad (51)$$

We will show that

$$\begin{aligned}e^{-L_0t}N(\mathbf{p}_1|\mathbf{k}_1)N(\mathbf{p}|\mathbf{k}_2)\rangle = \int\int \hat{\Gamma}(\mathbf{p}_1|\mathbf{p}_3|\mathbf{k}_1)\hat{\Gamma}(\mathbf{p}|\mathbf{p}_4|\mathbf{k}_2) \\ \times |N(\mathbf{p}_3|\mathbf{k}_1)N(\mathbf{p}_4|\mathbf{k}_2)\rangle\, d\, p^e\, d\, p_t \end{aligned} \quad (52)$$

We will exhibit the form of the time-dependent $\hat{\Gamma}$ shortly. The memory function matrix elements are

$$\begin{aligned}\langle N(\mathbf{p}'|\mathbf{k})|\hat{M}(t)|N(\mathbf{p}|\mathbf{k})\rangle = -\frac{i}{\Omega}\frac{\partial}{\partial p}\sum \mathbf{k}_1V(k_1)\int \hat{H}(\mathbf{k}_1|\mathbf{p}_1)\hat{\Gamma}(\mathbf{k}_2|\mathbf{p}|\mathbf{p}_2) \\ \times K(\mathbf{p}'\mathbf{p}_1\mathbf{p}_2|\mathbf{k}\mathbf{k}_1\mathbf{k}_2)d^3p_1\, d^3p_2 \end{aligned} \quad (53)$$

Here

$$\hat{H}(\mathbf{k}|\mathbf{p}) = \int \hat{\Gamma}(\mathbf{k}|\mathbf{y}|\mathbf{p})\, d^3y \quad (54)$$

The kernel K is the matrix element of PLQ. Thus

$$K(\mathbf{p}'\mathbf{p}_1\mathbf{p}_2|\mathbf{k}\mathbf{k}_1\mathbf{k}_2) = \langle N(\mathbf{p}'|\mathbf{k})|LQ|N(\mathbf{p}_1|\mathbf{k}_1)N(\mathbf{p}_2|\mathbf{k}_2)\rangle \quad (55)$$

To calculate $\hat{\Gamma}$, note that the bracket notation is inessential for functions to the right of operators. Then

$$e^{-L_0t}N(\mathbf{p}_1|\mathbf{k}_1)N(\mathbf{p}_2|\mathbf{k}_2) = \hat{N}(\mathbf{p}_1|\mathbf{k}_1)\hat{N}(\mathbf{p}_2|\mathbf{k}_2) \quad (56)$$

where

$$\hat{N}(\mathbf{p}|\mathbf{k}) = e^{-L_0 t} N(\mathbf{p}|\mathbf{k}) e^{L_0 t} \tag{57}$$

and $e^{L_0 t} \cdot 1 = 1$. (We use the caret to denote time dependence of the quantity involved.) $\hat{N}(\mathbf{p}|\mathbf{k})$ is computed from the equation

$$d\hat{N}/dt = -[L_0, \hat{N}] \tag{58}$$

involving the commutator of L_0 and $\hat{N}$. This yields

$$\frac{d\hat{N}}{dt}(\mathbf{p}|\mathbf{k}) = -\frac{i\mathbf{k}\cdot\mathbf{p}}{m}\hat{N}(\mathbf{p}|\mathbf{k}) - \frac{i\mathbf{k}\tilde{\Psi}}{\Omega}(k)\frac{N\,\partial\phi(p)}{\partial\mathbf{p}}\hat{\rho}(\mathbf{k}) \tag{59}$$

This is a modified Vlasov equation, which is to be solved with the condition that $\hat{N}(\mathbf{p}|\mathbf{k})$ at $t = 0$ equals $N(\mathbf{p}|\mathbf{k})$. The solution for $\hat{N}(\mathbf{p}|\mathbf{k})$ is given in terms of its Laplace transform $\bar{N}(\mathbf{p}|\mathbf{k}|s)$. It is

$$\bar{N}(\mathbf{p}|\mathbf{k}|s) = \left(s + \frac{i\mathbf{k}\mathbf{p}}{m}\right)^{-1}\left\{N(\mathbf{p}|\mathbf{k}) - \frac{i\mathbf{k}\tilde{\Psi}}{\Omega}(k)\frac{N\,\partial\phi(p)}{\partial\mathbf{p}} \times \frac{1}{\Delta(k|s)}\int \frac{N(\mathbf{p}'|\mathbf{k})}{s + (i\mathbf{k}\cdot\mathbf{p}'/m)}\,d^3p'\right\} \tag{60}$$

$$\Delta(k|s) = 1 + \frac{i\mathbf{k}\tilde{\Psi}(k)}{\Omega}N\int \frac{\partial\phi(x)}{\partial\mathbf{x}}\frac{1}{s + (i\mathbf{k}\cdot\mathbf{x}/m)}\,d^3\mathbf{x} \tag{61}$$

Everything is dependent on the single time-dependent function $\hat{H}(\mathbf{k}|\mathbf{p})$,

$$\hat{H}(\mathbf{k}|\mathbf{p}) = \frac{1}{2\pi i}\oint \frac{e^{st}}{\Delta(k|s)}\frac{1}{s + (i\mathbf{k}\cdot\mathbf{p}/m)}\,ds \tag{62}$$

Using a partial fraction decomposition in the solution for $\bar{N}(\mathbf{p}|\mathbf{k}|s)$, we obtain

$$\hat{\Gamma}(\mathbf{k}|\mathbf{p}|\mathbf{p}') = \delta(\mathbf{p} - \mathbf{p}')\exp(-i\mathbf{k}\cdot\mathbf{p}t/m) + \hat{\Lambda}(\mathbf{k}|\mathbf{p}|\mathbf{p}') \tag{63}$$

$$\hat{\Lambda}(\mathbf{k}|\mathbf{p}|\mathbf{p}') = -\frac{i\mathbf{k}\tilde{\Psi}(k)}{\Omega}\frac{N\,\partial\phi(p)}{\partial\mathbf{p}}\frac{\hat{H}(\mathbf{k}|\mathbf{p}') - \hat{H}(\mathbf{k}|\mathbf{p})}{(i\mathbf{k}/m)(\mathbf{p} - \mathbf{p}')} \tag{64}$$

To complete the calculation of the memory function, we must evaluate the kernel K. For $\mathbf{k} = 0$, the calculation is simple and we find

$$K(\mathbf{p}'\mathbf{p}_1\mathbf{p}_2|0, k_1, -k_1) \equiv \langle N(\mathbf{p}^1|0)|LQ|N(\mathbf{p}_1|\mathbf{k}_1)N(\mathbf{p}_2|-\mathbf{k}_1)\rangle = i\mathbf{k}_1 r(k_1)\phi(p_1)\phi(p_2)\frac{\partial}{\partial\mathbf{p}'}[\delta(\mathbf{p}' - \mathbf{p}_1) - \delta(\mathbf{p}' - \mathbf{p}_2)] \tag{65}$$

This involves only the equilibrium pair correlation function. For $\mathbf{k} \neq 0$, the

calculation is slightly more involved, and the result also involves the triplet correlation function.

$$r_3(\mathbf{k}_1|\mathbf{k}_2|-(\mathbf{k}_1+\mathbf{k}_2)) \equiv \int \Phi\, d\Gamma \sum_{\alpha\neq\beta\neq\gamma} \exp[i(\mathbf{k}_1\mathbf{q}_\alpha + \mathbf{k}_2\mathbf{q}_\beta - (\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{q}_\gamma] \tag{66}$$

We find, with $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$

$$\begin{aligned} K(\mathbf{p}'\mathbf{p}_1\mathbf{p}_2|\mathbf{k}\mathbf{k}_1\mathbf{k}_2) &= -i\frac{\phi_1\phi_2}{\theta}\frac{\partial}{\partial p}[\mathbf{k}_1 r(k_1)\,\delta(\mathbf{p}'-\mathbf{p}_1) + \mathbf{k}_2 r(k_2)\,\delta(\mathbf{p}'-\mathbf{p}_2)] \\ &\quad - \frac{i\mathbf{k}}{\theta}\frac{\phi_1\phi_2}{N+r(k)}\frac{\partial\phi(p')}{\partial\mathbf{p}'}[Nr_3(\mathbf{k}_1|\mathbf{k}_2|-\mathbf{k}) - r(k)\{r(k_1)+r(k_2)\}] \end{aligned} \tag{67}$$

With these results one obtains the memory function for dressed particles. We use the notation

$$\hat{I}(\mathbf{k}_1) \equiv \int \hat{H}(\mathbf{k}_1|\mathbf{p}_1)\phi(p_1)\, d^3p_1 \tag{68}$$

$$\hat{J}(\mathbf{k}_2|\mathbf{p}) \equiv \int \hat{\Gamma}(\mathbf{k}_2|\mathbf{p}|\mathbf{p}_2)\phi(p_2)\, d^3p_2 \tag{69}$$

Then

$$\begin{aligned} &\langle N(\mathbf{p}'|\mathbf{k})|\hat{M}(t)|N(\mathbf{p}|\mathbf{k})\rangle \\ &\quad = -\frac{1}{\theta}\frac{\partial}{\partial\mathbf{p}}\sum\frac{\mathbf{k}_1\tilde{V}(k_1)}{\Omega}\frac{\partial}{\partial\mathbf{p}}\{\mathbf{k}_1 r(k_1)\hat{I}(\mathbf{k}_1)\hat{\Gamma}(\mathbf{k}_2|\mathbf{p}|\mathbf{p}')\phi(p') \\ &\qquad + \mathbf{k}_2 r(k_2)\hat{H}(\mathbf{k}_1|\mathbf{p}')\hat{J}(\mathbf{k}_2|\mathbf{p})\phi(p')\} \\ &\qquad + \frac{1}{\theta}\frac{\mathbf{k}}{N+r(k)}\frac{\partial\phi(p')}{\partial\mathbf{p}'}\frac{\partial}{\partial\mathbf{p}}\sum_k\frac{\mathbf{k}_1 V(k_1)}{\Omega}\hat{I}(\mathbf{k}_1)\hat{J}(\mathbf{k}_2|\mathbf{p}) \\ &\qquad \times\{Nr_3(\mathbf{k}_1|\mathbf{k}_2|-\mathbf{k}) - r(\mathbf{k})[r(\mathbf{k}_1)+r(\mathbf{k}_2)]\} \end{aligned} \tag{70}$$

To conclude this section, we note that our evaluation of the memory function in the time domain is similar to Dupree's approach[17] to plasmas, where, however, we do not make an adiabatic assumption. The calculation can also be carried out in the Laplace transform domain using Guernsey's application of the theory of singular integral equations. The time domain approach is more direct and involves less sophisticated mathematics.

5. KINETIC EQUATIONS

5.1. Spatially Homogeneous Processes

Introduce the correlation function

$$\bar{D}_0(\mathbf{p}|\mathbf{p}') = \int Z_0(\mathbf{p}|\mathbf{p}'')\, d^3p''\, \bar{C}_0(\mathbf{p}''|\mathbf{p}') \tag{71}$$

The zero subscript indicates that quantities are for zero wave vector. Given $\bar{D}_0$, we can easily find $\bar{C}_0$. The relation is

$$\bar{C}_0(\mathbf{p}|\mathbf{p}') = N\phi(p)\{\bar{D}_0(\mathbf{p}|\mathbf{p}') + (N-1)\int \phi_1 \bar{D}_0(\mathbf{p}_1|\mathbf{p}')\, d^3p_1\} \tag{72}$$

The equation for $\bar{D}_0$ is

$$S\bar{D}_0(\mathbf{p}'|\mathbf{p}) + \frac{1}{N\phi(p')}\int \langle \mathbf{p}'|\bar{M}_0|\mathbf{p}''\rangle\, d^3p''\, \bar{D}_0(\mathbf{p}''|\mathbf{p}) = \delta(\mathbf{p}-\mathbf{p}') \tag{73}$$

Using the results already obtained, we have, in the time domain,

$$\langle \mathbf{p}'|\hat{M}_0|\mathbf{p}\rangle = \sum_k \frac{r(k)V(k)}{\theta\Omega} k_\mu k_\nu \frac{\partial}{\partial p_\mu'}\frac{\partial}{\partial p_\nu} \times \left\{\phi(p')\left[\hat{H}(\mathbf{k}|\mathbf{p}')\int \hat{\Gamma}(-\mathbf{k}|\mathbf{p}|\mathbf{y})\phi(y)\, d^3y - \hat{I}(\mathbf{k})\hat{\Gamma}(-\mathbf{k}|\mathbf{p}|\mathbf{p}')\right]\right\} \tag{74}$$

We have $\hat{\Gamma}(\mathbf{k}|\mathbf{p}|\mathbf{p}') = \delta(\mathbf{p}-\mathbf{p}')\exp(-i\mathbf{k}\mathbf{p}t/m) + \hat{\Lambda}(\mathbf{k}|\mathbf{p}|\mathbf{p}')$ with $\hat{\Lambda}$ proportional to $\tilde{\Psi}(k)$.

It is important to separate the delta function part of $\hat{\Gamma}$; we introduce

$$\hat{M}_0(t) = \hat{M}_0{}^0(t) + \hat{M}_0{}'(t) \tag{75}$$

where $\hat{M}_0{}'$ is the same as $\hat{M}^0$, with $\hat{\Gamma}$ replaced by $\hat{\Lambda}$, leaving the functions $\hat{H}$ and $\hat{I}$ unaltered. We now define

$$\hat{G}_{\mu\nu}(\mathbf{p}'|\mathbf{p}) = \sum [r(k)/\theta\Omega]\tilde{V}(k)k_\mu k_\nu H(\mathbf{k}|\mathbf{p}')\exp(i\mathbf{k}\mathbf{p}t/m) \tag{76}$$

$$\hat{R}_{\mu\nu}(\mathbf{p}') = \int \hat{G}_{\mu\nu}(\mathbf{y}|\mathbf{p}')\phi(\mathbf{y})\, d^3y \tag{77}$$

$\bar{G}_{\mu\nu}(\mathbf{p}'|\mathbf{p})$ and $\bar{R}_{\mu\nu}(\mathbf{p}')$ are the Laplace transforms of these functions. We then have

$$\langle \mathbf{p}'|\hat{M}_0{}^0(t)|\mathbf{p}\rangle = \frac{\partial}{\partial p_\mu'}\frac{\partial}{\partial p_\nu}\{\phi(p')[\hat{G}_{\mu\nu}(\mathbf{p}'|\mathbf{p}) - \delta(\mathbf{p}-\mathbf{p}')\hat{R}_{\mu\nu}(\mathbf{p}')]\} \tag{78}$$

The equation for the correlation function is

$$S\bar{D}_0(\mathbf{p}'|\mathbf{p}) + \frac{1}{N\phi(p')}\frac{\partial}{\partial p_{\mu}'}\left\{\phi(p')\bar{R}_{\mu\nu}(\mathbf{p}')\frac{\partial \bar{D}_0}{\partial p_{\nu}'}(\mathbf{p}'|\mathbf{p})\right\}$$

$$-\frac{1}{N\phi(p')}\frac{\partial}{\partial p_{\mu}'}\left[\phi(p')\int \bar{G}_{\mu\nu}(\mathbf{p}'|\mathbf{p}'')\,d^3p''\,\frac{\partial \bar{D}_0}{\partial p_{\nu}''}(\mathbf{p}''|\mathbf{p})\right]$$

$$+\frac{1}{N\phi(p')}\int \langle \mathbf{p}'|\bar{M}'(s)|\mathbf{p}''\rangle\, d^3p''\,\bar{D}_0(\mathbf{p}''|\mathbf{p}) = \delta(\mathbf{p}'-\mathbf{p}) \qquad (79)$$

The first term has differential operators that arise from the delta function in $\hat{M}^0$. The memory function $\bar{M}'(s)$ has an extra factor $\bar{\Psi}(k)$ as compared with $\bar{M}^0(s)$. Thus it is of higher order in the plasma parameter for Coulomb systems and of higher order in the density for systems with short-range forces.

Let us now compare the memory function $\bar{M}^0(s)$ with the weak coupling memory function of Section 3. We first compare the terms involving the delta functions. The key factor is $\bar{B}_k(p)$ for the weak coupling theory. It is to be compared with

$$\int_0^{\infty} [\exp(-st)]\hat{I}(\mathbf{k}_1)\exp(i\mathbf{k}_1\mathbf{p}'t/m)\,dt$$

Now

$$\hat{I}(k) = \frac{1}{2\pi i}\int e^{st}\frac{E(k|s)}{\Delta(k|s)}, \qquad E(k|s) = \int \frac{\phi(p)\,d^3p}{s + (ikp/m)} \qquad (80)$$

Thus if we neglect screening effects and replace Δ by unity, we find $\hat{I}(k) \to \int [\exp(-i\mathbf{k}\eta t/m)]\phi(\eta)\,d^3\eta$. Inserting this expression, we find precisely the weak coupling $\bar{B}_k(\mathbf{p})$.

The other term in the weak coupling memory function involves the structure $\{s + i[\mathbf{k}(\mathbf{p}'-\mathbf{p})/m]\}^{-1}$. It arises again when one makes the approximation of neglecting the screening so that the function

$$\hat{H}(\mathbf{k}|\mathbf{p}) \to \exp(-i\mathbf{k}\mathbf{p}t/m)$$

For long-range forces, the inclusion of the factor $\Delta(k_1|s)$ is essential to avoid long-wavelength divergences. When the pair distribution $r(k)$ is evaluated in the Debye–Hückel approximation, the screening is the standard plasma function that occurs in the Balescu–Lenard–Guernsey treatment. The only difference is that the presence of the exact $r(k)$ in the memory operator $\hat{M}^{(0)}(t)$ also yields a sensible behavior at short distances. The additional contributions to the memory operator represented by $\hat{M}^{(1)}(t)$ of course give additional terms not contained in either the usual weak coupling or plasma theories.

In a sense the usual plasma theory is richer in structure than the weak coupling theory or theory for gases with strong short-range forces. It includes screening and "backflow" effects in the description of the collisions between a pair of particles. These effects are also important in macroscopic hydrodynamics when one studies the effective interaction between two macroscopic bodies moving in the fluid. One expects that there is a residue that persists even when the two bodies are of atomic size. The theory that has been presented yields a description of these effects.

Of course, as mentioned in the introduction, we do not have a really proper theory of liquids or even of low-density gases. No attempt has been made to provide a proper account of the binary collisions between bare particles.

5.2. Spatially Inhomogeneous Processes

We now set down the kinetic equation for nonzero wave vector correlation functions. Introduce

$$\bar{D}(\mathbf{pk}|\mathbf{p'k}) \equiv \int Z(\mathbf{pk}|\mathbf{p}_1\mathbf{k})\, d^3p_1\, \bar{C}(\mathbf{p}_1\mathbf{k}|\mathbf{p'k}) \tag{81}$$

Then

$$\bar{C}(\mathbf{pk}|\mathbf{p'k}) = N\phi(p)\bar{D}(\mathbf{pk}|\mathbf{p'k}) + r(k)\phi(p)\int \phi_1 \bar{D}(\mathbf{p}_1\mathbf{k}|\mathbf{p'k})\, d^3p_1 \tag{82}$$

We will not write the dependence on k explicitly in those cases where this does not lead to confusion.

The equation obeyed by $\bar{D}$ is

$$\left(s + i\frac{\mathbf{k}\cdot\mathbf{p}}{m}\right)\bar{D}(\mathbf{p}|\mathbf{p'}) - \frac{r(k)}{N+r}\int \frac{i\mathbf{k}\cdot\mathbf{p}''}{m}\, d^3p''\, \bar{D}(\mathbf{p}''|\mathbf{p'})$$
$$+ \frac{1}{N\phi(p)}\int \langle \mathbf{p}|\bar{M}(s)|\mathbf{p}''\rangle\, d^3\mathbf{p}''\, \bar{D}(\mathbf{p}''|\mathbf{p'}) = \delta(\mathbf{p} - \mathbf{p'}) \tag{83}$$

Here we have simplified the equation by using the fact that

$$\int d^3p_2\, \langle N(\mathbf{p}_2|\mathbf{k})|\bar{M}(s)|N(\mathbf{p}''\mid \mathbf{k})\rangle = 0 \tag{84}$$

We must again pay special attention to the delta function contribution of $\hat{\Gamma}$. Let

$$\langle N(\mathbf{p'}|\mathbf{k})|\hat{M}^0(t)|N(\mathbf{p}|\mathbf{k})\rangle = -\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial p_\nu'}[\delta(\mathbf{p} - \mathbf{p'})\hat{R}_{\mu\nu}(\mathbf{k}|\mathbf{p})\phi(p')$$
$$- \phi(p)\phi(p')\hat{G}_{\mu\nu}(\mathbf{k}|\mathbf{p'}|\mathbf{p})]$$
$$- \frac{k_\mu}{N + r(k)}\frac{\partial\phi(p')}{\partial p_\mu'}\frac{\partial}{\partial p_\nu}[\hat{T}_\nu(\mathbf{k}|\mathbf{p})\phi(p)] \tag{85}$$

Here

$$\hat{G}_{\mu\nu}(\mathbf{k}|\mathbf{p}'|\mathbf{p}) = -\sum_{k_1} (k_{1\mu}k_{1\nu}/\theta\Omega)\tilde{V}(k_1)r(k_2)\hat{H}(\mathbf{k}_1|\mathbf{p}')\exp(-i\mathbf{k}_2\mathbf{p}t/m) \quad (86)$$

$$\hat{R}_{\mu\nu}(\mathbf{k}|\mathbf{p}) = [\exp(-i\mathbf{k}\cdot\mathbf{p}t/m)]\hat{R}_{\mu\nu}(\mathbf{p}), \qquad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k} \quad (87)$$

$$\hat{T}_\nu(\mathbf{k}|\mathbf{p}) = \sum_{k_1} (k_{1\nu}/\theta\Omega)\tilde{V}(k_1)\hat{I}(\mathbf{k}_1)\exp(i\mathbf{k}_2\mathbf{p}t/m)$$
$$\times \{Nr_3(\mathbf{k}_1|\mathbf{k}_2|-\mathbf{k}) - r(k)[r(k_1) + r(k_2)]\} \quad (88)$$

The residual part of the memory operator $\hat{M}^{(1)}(t)$ is defined as Eq. (70) with $\hat{\Gamma}(\mathbf{k}_2|\mathbf{p}|\mathbf{p}')$ replaced by $\hat{\Lambda}(\mathbf{k}_2|\mathbf{p}|\mathbf{p}')$ and $\hat{N}(\mathbf{k}_2|\mathbf{p})$ replaced by

$$\int \hat{\Lambda}(\mathbf{k}_2|\mathbf{p}|\mathbf{p}_2)\phi_2\, d^3p_2$$

The complete kinetic equation for inhomogeneous processes follows by inserting the Laplace transforms of the memory function, which we have found in the time domain, into Eq. (83). Again, it is easy to recover standard weak coupling and plasma kinetic equations by making further approximations.

6. SUMMARY

It is not our intention here to make a deeper study of the kinetic equations arrived at in the present paper. It is easy to write more accurate kinetic equations using, for example, the two- or three-body additive approximation of the present author or Mazenko's truncation scheme. The price one must pay for accuracy is a much more complicated mathematical structure. If one is to have an understandable unified theory of fluids, the more accurate theories have to be rearranged and analyzed into component features. Our goal has been to show how the very important screening effects that were first given a clear mathematical expression in plasma physics can be embedded in a unified theory of fluids. The use of projection operators and a judicious choice of an "unperturbed" Liouville operator make possible the isolation of the screening effects from a more complete theory.

The theory of time-dependent correlation functions has been studied by a variety of different analytical techniques in recent years. One of our aims has been to show that it is not necessary to be committed completely to a single technique.

There is a great deal of flexibility in the choice of the function space defining the projection operator P. It is not necessary to use the scheme of Ref. 6, i.e., body additive functions. For example, there are well-defined function spaces in between the one- and two-body additive spaces, such as the

A, B, and C sectors of Ref. 7. Another example is a modification of the one-body additive space so that the kinetic energy density is replaced by the total energy density. Bergeron[18] and the author have shown that this choice makes possible a lucid discussion of the hydrodynamic limit.[19] A similar modification of the two-body additive theory permits one to give an extended microscopic mode–mode coupling theory along the lines pioneered in Ref. 4. The utility of interpolatory projection operators is apparent, since a very large function space leads to intractable equations which must, in any case, be further analyzed.

In the present paper we have studied only one aspect of what can be done to approximate the memory kernel that takes into account effects of the Q space orthogonal to the chosen P space. Of course, one always has available diagrammatic perturbation theory, cluster analysis, etc., to analyze the propagator contained in the memory operator. Our stress has not been on these techniques, which usually involve analysis in terms of some small parameter. Instead, using the functional formulation, we have shown that the freedom to choose the unperturbed Liouville operator L_0 may be used to obtain significant results without much effort and without commitment to a small-parameter analysis. Of course, to complete the theory, one should continue with diagrammatic analysis of the effects of the perturbed operator L_1 to find the magnitude of correction terms.

This procedure makes possible a more unified theory of fluids and plasmas in which most of the differences lie in the different equilibrium correlation functions. However, in any practical approximation there may well remain significant differences that require taking account of the dynamical effects of L_1.

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